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DQM FOR VIBRATION ANALYSIS OF COMPOSITE THIN-WALLED CURVED BEAMS

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1. INTRODUCTION

Thin-walled curved beams are extensively used in engineering applications when requirements of weight saving are of primary importance. This is especially true when composite materials are used [1]. For the analysis of isotropic thin-walled beams, Vlasov's theory was applied successfully in several applications [2].

Kang *et al.* [3] have shown that the differential quadrature method (DQM) gives very accurate results for natural frequencies of horizontally curved thin-walled beams which are modelled by Vlasov's theory. More recently, Chen [4] applied successfully a variant of the DQM for analyzing the free vibration of straight thin-walled beams made of isotropic materials.

In this letter, natural frequencies for the out-of-plane vibration of composite thin-walled curved beams are determined by means of the differential quadrature method.

The structural model used in this article takes into account the shear flexibility effect (warping and bending shear) that may be of great importance for composite beams [5].

The effectiveness of the DQM is demonstrated by comparing the present results with those obtained by analytical methods. On the other hand, the influence of the shear flexibility is analyzed by comparing the frequencies obtained with the present theory and those determined by Vlasov's model (i.e., neglecting the shear effect).

2. STRUCTURAL MODEL

The structural model used in this study was recently developed [5]. Its main assumptions are as follows: (1) the cross-section is rigid in its own plane; (2) all components of stresses, excepting σ_x and τ_{xs}, are neglected; (3) the laminate sequence is assumed to be symmetric and balanced or specially orthotropic.

Under such assumptions the governing equations are given by

$$-\frac{\partial Q_y}{\partial x} + \rho A\left(\frac{\partial^2 v_s}{\partial t^2} - z_0\frac{\partial^2 \phi}{\partial t^2}\right) = 0, \tag{1a}$$

$$\frac{\partial M_z}{\partial x} - Q_y - \frac{1}{R}\left(\frac{\partial B}{\partial x} - T_{sv}\right) + \rho\left(I_z\frac{\partial^2 \theta_z}{\partial t^2} + \frac{C_w}{R}\frac{\partial^2 \theta}{\partial t^2}\right) = 0, \tag{1b}$$

$$-\frac{\partial B}{\partial x} - T_w + \rho C_w\left(\frac{\partial^2 \theta}{\partial t^2} + \frac{1}{R}\frac{\partial^2 \theta_z}{\partial t^2}\right) = 0, \tag{1c}$$

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$$-\frac{\partial T_w}{\partial x}-\frac{\partial T_{sv}}{\partial x}+\frac{M_z}{R}-\rho A z_0\frac{\partial^2 v_s}{\partial t^2}+\rho I_s\frac{\partial^2\phi}{\partial t^2}=0, \tag{1d}$$

in which

$$M_z=-E^*I_z\left(\frac{\partial\theta_z}{\partial x}-\frac{\phi}{R}\right), \tag{2a}$$

$$Q_y=G^*\left[S_{yy}\left(\frac{\partial v_s}{\partial x}-\theta_z\right)+S_{y\omega}\left(\frac{\partial\phi}{\partial x}-\theta\right)\right], \tag{2b}$$

$$T_{SV}=G^{**}J\left(\frac{\partial\phi}{\partial x}+\frac{\theta_z}{R}\right),\qquad B=E^*C_W\left(\frac{\partial\theta}{\partial x}+\frac{1}{R}\frac{\partial\theta_z}{\partial x}\right), \tag{2c, d}$$

$$T_w=G^*\left[S_{y\omega}\left(\frac{\partial v_s}{\partial x}-\theta_z\right)+S_{\omega\omega}\left(\frac{\partial\phi}{\partial x}-\theta\right)\right]. \tag{2e}$$

In the above equations, v_s is the transverse displacement at the shear center, ϕ is the rotation about the x-axis, θ_z is the bending rotation, θ is a measure of warping, Q_y is the shear force, M_z is the bending moment, B is the bimoment, T_{sv} is the Saint–Venant torsional moment, T_w is the flexural torsional moment, I_z is the inertia moment, C_w is the warping constant, J is the torsion constant, z_0 is the co-ordinate of the centroid with respect to the shear center, ρ is the density of the material, A is the cross-sectional area, I_s is the polar inertia moment with respect to the shear center and R is the radius of curvature.

The S_{ij}'s appearing in expressions (2) are shear coefficients whose definitions are given by

$$[S_{fg}]=\begin{bmatrix}\int_0^m \frac{e\lambda_z^2}{I_z^2}\,\mathrm{d}s & \int_0^m \frac{e\lambda_z\lambda_\omega}{C_wI_z}\,\mathrm{d}s\\ \int_0^m \frac{e\lambda_z\lambda_\omega}{C_wI_z}\,\mathrm{d}s & \int_0^m \frac{e\lambda_\omega^2}{C_w^2}\,\mathrm{d}s\end{bmatrix}^{-1},\quad f,g=y,\omega, \tag{3}$$

where

$$\lambda_z(s)=\int_0^s Y(s)\,\mathrm{d}s,\quad \lambda_\omega(s)=\int_0^s \omega(s)\,\mathrm{d}s. \tag{4}$$

On the other hand, the material properties given by E^*, G^* and G^{**} coincide with the laminate moduli E_x, G_{xy} and G^b_{xy} defined in reference [1, p. 156].

3. DIFFERENTIAL QUADRATURE METHOD

According to DQM [6, 7], the domain is divided into N discrete points. Then the derivatives of a function at a discrete point are expressed in terms of the function values at all discrete points:

$$\frac{\partial^n f(x_i)}{\partial x^n}=\sum_{k=1}^{N} C^n_{ik}f(x_k), \tag{5}$$

where the weighting coefficients C^n_{ik} may be easily obtained by means of the recurrence formulas of Shu and Chen [7].

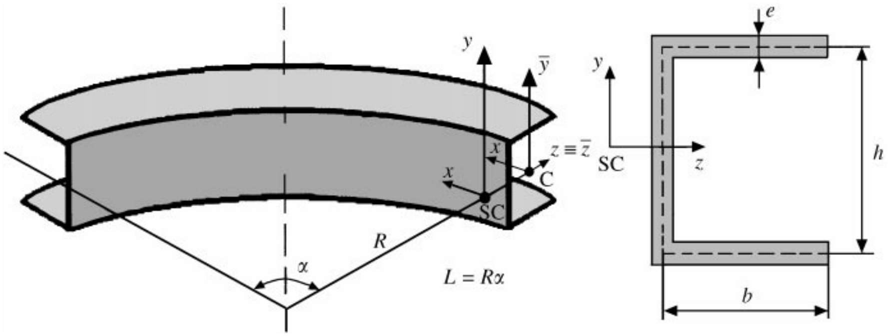

Figure 1. Structural member and cross-section: (SC) shear center, (C) cross-section centroid; $e = 0{\cdot}03$ m, $h = 0{\cdot}60$ m, $b = 0{\cdot}60$ m.

Operator (5) is applied for each of the derivatives arising in equations (1, 2) and in the corresponding boundary conditions. Thus, the differential system is approximately transformed into an algebraic one, whose unknowns are given by the discretized generalized displacements

$$\mathbf{u}_i = [v_i\ \theta_{zi}\ \theta_i\ \phi_i]^{\mathrm{T}}. \tag{6}$$

To obtain the free vibration frequencies, normal mode solutions are assumed in the form

$$\mathbf{u}_{\mathrm{i}} = \mathbf{U}(x_i)\sin(2\pi f t), \tag{7}$$

where f is the natural frequency (in Hz).

After eliminating the end unknowns from the appropriate boundary conditions, one obtains the general eigenvalue problem of the form

$$[\mathbf{A} - \lambda\mathbf{B}]\{\mathbf{U}\} = 0. \tag{8}$$

In the above equation, $\lambda = (2\pi f)^2$.

4. NUMERICAL EXAMPLES

The U-section beam shown in Figure 1 is considered. The analyzed material is graphite-epoxy (AS4/3501) whose properties are given by $E_1 = 144$ GPa, $E_2 = 9{\cdot}65$ GPa, $G_{12} = 4{\cdot}14$ GPa, $\nu_{12} = 0{\cdot}3$, $\rho = 1389$ Kg/m^3. In the present study the shifted Chebyshev–Gauss–Lobatto points are adopted as the mesh collocation points. A value of $R = 12$ m is adopted for the radius of curvature.

Table 1 shows the first six frequencies for a simply supported beam ($v_s = M_z = \phi = B = 0$, at $x = 0, L$). The present results are compared with exact analytical results obtained in reference [5]. As it may be seen, the values are practically coincident. Table 2 shows a convergence analysis for clamped beams ($v_s = \theta_z = \phi = \theta = 0$, at $x = 0, L$). It is possible to see that nine collocation points are sufficient for obtaining good accuracy. Finally, Table 3 shows the first six frequencies for a clamped beam. The present results are compared with Vlasov's results (neglecting shear flexibility, warping inertia and rotatory inertia). As it may be seen, the shear effect is noticeable for the laminates (0/0/0/0) and (0/90/90/0).

TABLE 1

Comparisons with DQM and analytical solutions for shear flexible U-section beams with simply supported ends ($N = 21$)

Laminate	L(m)	Model	f_1	f_2	f_3	f_4	f_5	f_6
		[I]	2·89	22·42	53·84	89·25	114·31	125·47
(a)	12	[II]	2·89	22·42	53·84	89·25	114·31	125·47
		[III]	**0·00**	**0·00**	**0·00**	**0·00**	**0·00**	**0·00**
(0/0/0/0)		[I]	22·42	89·25	148·22	161·48	232·03	251·83
	6	[II]	22·42	89·25	148·22	161·48	232·03	251·83
		[III]	**0·00**	**0·00**	**0·00**	**0·00**	**0·00**	**0·00**
		[I]	2·20	17·53	44·13	76·37	93·58	111·10
(b)	12	[II]	2·20	17·53	44·12	76·37	93·57	111·09
		[III]	**0·01**	**0·01**	**0·01**	**0·01**	**0·01**	**0·01**
(0/90/90/0)		[I]	17·53	76·37	134·74	146·71	218·02	243·84
	6	[II]	17·53	76·37	134·73	146·70	218·01	243·83
		[III]	**0·01**	**0·01**	**0·01**	**0·01**	**0·00**	**0·00**
		[I]	1·43	9·44	24·16	44·72	48·56	70·55
(c)	12	[II]	1·42	9·44	24·15	44·71	48·56	70·53
		[III]	**0·09**	**0·04**	**0·03**	**0·02**	**0·01**	**0·02**
(45/ − 45/ − 45/45)		[I]	9·44	44·72	90·90	101·14	174·55	234·39
	6	[II]	9·44	44·71	90·88	101·11	174·50	234·26
		[III]	**0·04**	**0·02**	**0·02**	**0·02**	**0·03**	**0·06**

Note: [I] DQM, [II] Analytical Solution [5], [III] Percentage difference $\varepsilon = 100\,|f_{[II]} - f_{[I]}|/f_{[II]}$.

TABLE 2

Convergence analysis for clamped U-section beams

Laminate	(a) 0/0/0/0				(b) 0/90/90/0			
L (m)	12		6		12		6	
N	f_1	f_2	f_1	f_2	f_1	f_2	f_1	f_2
5	26·51	62·95	59·59	126·21	22·02	35·40	51·01	124·03
7	23·46	41·33	58·51	103·90	19·48	33·64	49·85	96·02
9	23·37	40·02	58·52	103·53	19·40	33·64	49·86	95·35
11	23·37	40·01	58·52	103·53	19·40	33·64	49·86	95·34
13	23·37	40·01	58·52	103·53	19·40	33·64	49·86	95·34
15	23·37	40·01	58·52	103·53	19·40	33·64	49·86	95·34

5. CONCLUSIONS

In this paper, DQM was applied successfully for determining vibration frequencies of composite thin-walled curved beams. From the numerical results, it may be concluded that the shear effect may be of great importance for certain types of laminates.

TABLE 3

Flexural-torsional frequencies of composite U-section beams with clamped ends, study of the shear effect ($N = 21$)

Laminate	L(m)	Model	f_1	f_2	f_3	f_4	f_5	f_6
		[I]	22·37	40·01	66·53	97·41	117·03	131·88
(a)	12	[II]	32·90	58·08	108·55	182·02	188·75	276·39
		[III]	**28·98**	**31·11**	**38·71**	**46·48**	**38·00**	**52·28**
(0/0/0/0)		[I]	58·52	103·53	151·45	170·98	234·96	253·86
	6	[II]	89·63	230·06	454·88	528·96	760·80	1142·69
		[III]	**34·71**	**55·00**	**66·71**	**67·68**	**69·12**	**77·78**
		[I]	19·40	33·64	57·98	87·00	94·27	119·29
(b)	12	[II]	24·21	42·67	79·37	133·07	138·16	202·00
		[III]	**19·87**	**21·16**	**26·95**	**34·62**	**31·77**	**40·95**
(0/90/90/0)		[I]	49·86	95·34	136·66	158·99	223·86	245·92
	6	[II]	65·38	168·31	332·34	386·30	555·85	834·78
		[III]	**23·74**	**43·35**	**58·88**	**58·84**	**59·73**	**70·54**
		[I]	10·65	19·19	35·10	57·29	57·48	84·73
(c)	12	[II]	10·87	19·63	36·18	60·10	61·31	90·67
		[III]	**1·96**	**2·20**	**2·98**	**4·67**	**6·25**	**6·54**
(45/ − 45/ − 45/45)		[I]	28·85	70·24	131·33	134·34	208·32	282·71
	6	[II]	29·63	75·46	148·17	171·28	247·10	370·45
		[III]	**2·66**	**6·93**	**11·36**	**21·57**	**15·70**	**23·68**

Note: [I] Shear flexible model, [II] Vlasov-type model, [III] Percentage difference: $\varepsilon = 100\ [f_{[II]} - f_{[I]}]/f_{[II]}$.

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